Automated Reasoning on Consistency Models for Replicated Data Systems with MONA

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12^{th} FRIDA Workshop – October 27, 2025

Context

Consistency models

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Consistency models

Linearizability: every operation appears to take place atomically, in some order, consistent with the real-time ordering of those operations

Eventual consistency: all replicas converge to the same value eventually

Read-your-writes: each process always read its latest write

Monotonic reads: once a process has read a value of a data item, its future reads will never return an older value

Problems

Problem 1: Given an implementation of a replicated data system, can we formally and fully automatically verify that it satisfies a specific consistency model?

Problem 2: Given a distributed application that uses a (black-box) replicated data system, and assuming this system conforms to a given consistency model, can we formally and fully automatically verify that the application behaves correctly (e.g., is functional or safe)?

More generally:

How can we formally and automatically reason about consistency models in replicated data systems?

Table of Contents

Histories

We define $\mathcal{H}(\mathbb{P}, \mathbb{T}, \mathbb{O}, \mathbb{V})$ as the set of all well-defined finite histories.

- $\mathbb{P}$ is the **finite** set of processes
- $\mathbb{T} = \{read, write\}$ is the set of operation types
- $\mathbb{O}$ is the **finite** set of objects
- $\mathbb{V}$ is the **finite** set of values

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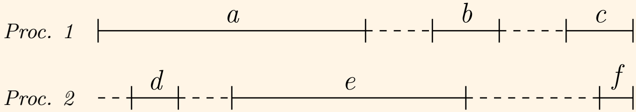
A history $H \in \mathcal{H}(\mathbb{P}, \mathbb{T}, \mathbb{O}, \mathbb{V})$ is a set of **operations**; each of its operations having attributes drawn from the sets above.

Histories



The MONA tool

* an automatic verification tool that analyzes logical formulas
* in particular, formulas of a fragment of **weak Monadic Second Order logic (MSO)**
* it translates MSO formulas into **finite**-state automata

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MSO logic allows $\forall x, \forall X, \exists x, \exists X, P(x), P(X), P(X, Y), ...$

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Automata's inputs are **finite** words of bit vectors

$$\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\cdots\begin{bmatrix}0\\0\end{bmatrix}$$

MSO logic of words of bit vectors

- A vector is called a **position**
- A row is called a **set of positions**

$$
\begin{aligned}
\psi := \; & \forall x.\psi \\
\mid \; & \exists x.\psi \\
\mid \; & \forall X.\psi \\
\mid \; & \exists X.\psi \\
\mid \; & x = \mathrm{succ}(y) \quad \text{with } \mathrm{succ}(y) := y + 1 \\
\mid \; & x = y \quad \text{equality of positions} \\
\mid \; & x \in X \quad \text{position } x \text{ is in the set } X \\
\mid \; & \psi \wedge \psi \mid \psi \vee \psi \mid \neg\psi \mid (\psi)
\end{aligned}
$$

Motivation

Theorem

The satisfiability of MSO formulas over finite histories is decidable

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Theorem
The satisfiability of MSO formulas over finite histories is decidable

Extending the result to infinite, non Zeno histories, seems easy

Monadic second-order logic of histories

Let a and b be some operations.

$$
\begin{aligned}
\phi := \; & a.proc = b.proc \quad \text{with } a.proc, b.proc \in \mathbb{P} \\
\mid \; & a.type = b.type \quad \text{with } a.proc, b.proc \in \mathbb{T} \\
\mid \; & a.obj = b.obj \quad \text{with } a.proc, b.proc \in \mathbb{P} \\
\mid \; & a.ival = b.ival \\
\mid \; & a.oval = b.oval \quad \text{with } a.ival, b.ival, a.ival, b.ival \in \mathbb{V} \\
\mid \; & t < t \\
\mid \; & \forall a.\phi \\
\mid \; & \exists a.\phi \\
\mid \; & \forall A.\phi \\
\mid \; & \exists A.\phi \\
\mid \; & \phi \wedge \phi \mid \phi \vee \phi \mid \neg\phi \mid (\phi)
\end{aligned}
$$

$$
\begin{aligned}
t := \; & a.stime \\
\mid \; & a.rtime
\end{aligned}
$$

Monadic second-order logic of histories

Arbitration: $a \xrightarrow{ar} b$ denotes that operation a is considered to be done before operation b

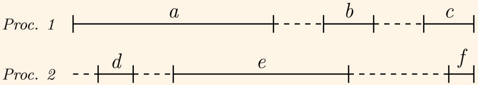
Visibility: $a \xrightarrow{vis} b$ denotes that the effects of operation a are visible to the client performing b

The MONA tool for histories

MONA handles **discrete time**, which can be seen as *snapshots*

$$\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix}\dots\begin{bmatrix}0\\0\end{bmatrix}$$

Histories represent **continuous time**, which can be seen as a *timeline*



Outline


Traces of executions called *Histories*
Input: words of bit vectors
Consistency verification *(with the MONA tool)*

Outline

Traces of executions called *Histories*

Histories as words of bit vectors

Input: words of bit vectors

↓

Consistency verification *(with the MONA tool)*

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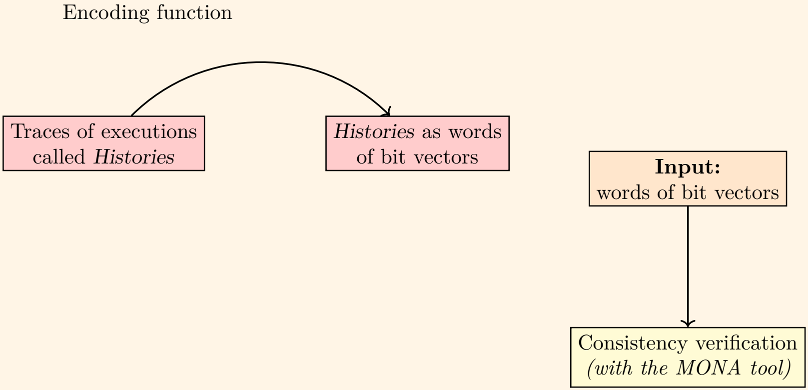
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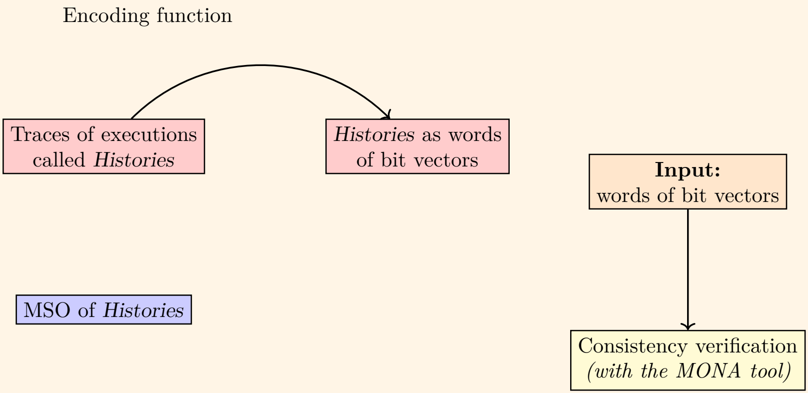
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MSO of *Histories*
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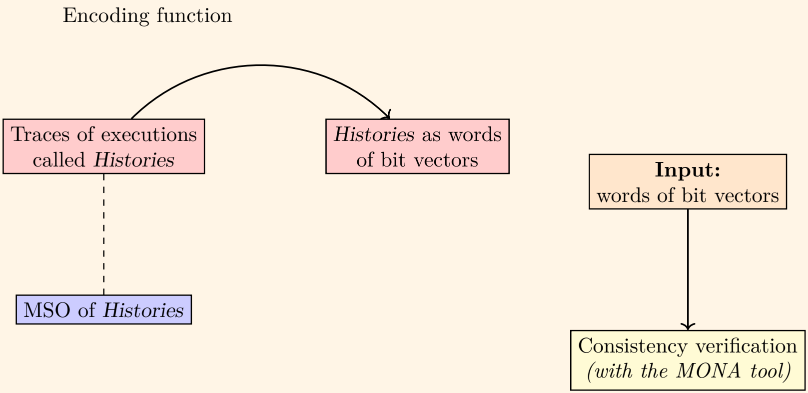
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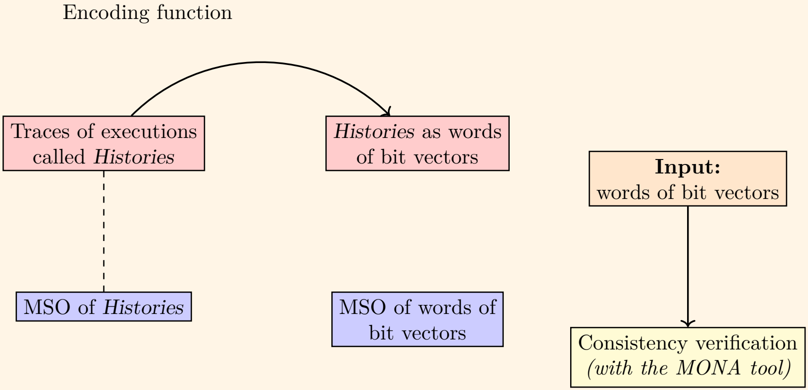
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MSO of *Histories*
MSO of words of bit vectors
Consistency verification (with the MONA tool)

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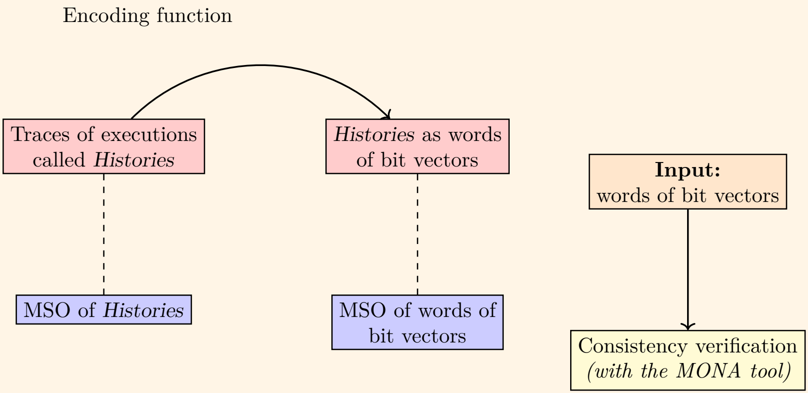
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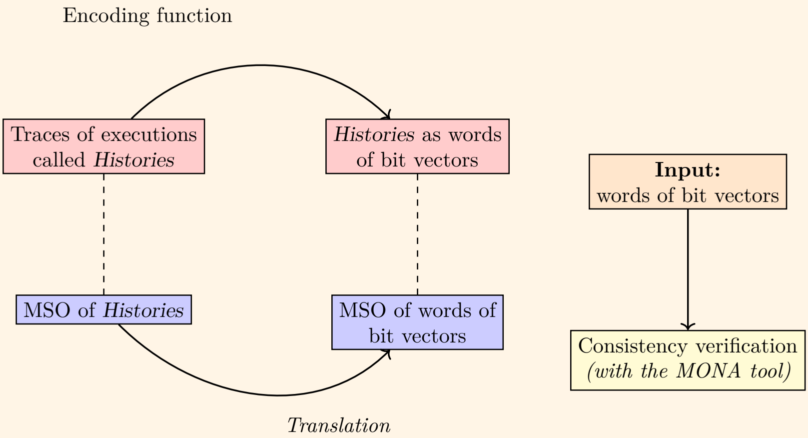
Encoding function
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MSO of *Histories*
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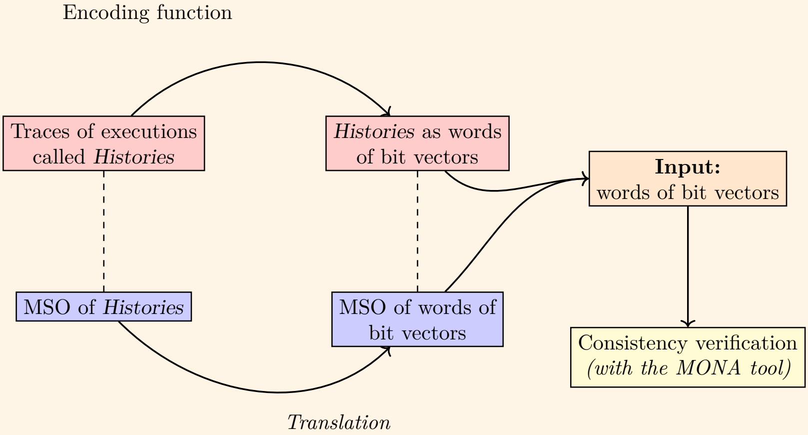
Translation
Input: words of bit vectors
Consistency verification *(with the MONA tool)*

Outline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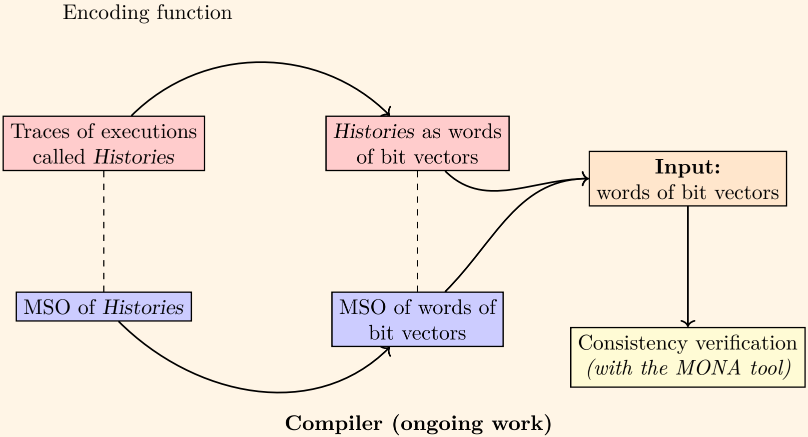
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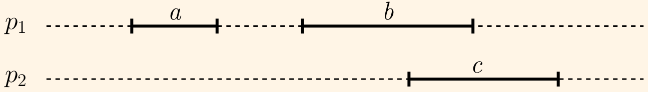
Histories as words of bit vectors
Input: words of bit vectors
MSO of *Histories*
MSO of words of bit vectors
Consistency verification *(with the MONA tool)*
Compiler (ongoing work)

Vector structure

$$\begin{bmatrix} p_1 \\ \vdots \\ p_n \\ t \\ v_1 \\ \vdots \\ v_k \\ o_1 \\ \vdots \\ o_\ell \\ \alpha_1 \\ \vdots \\ \alpha_a \\ \nu_1 \\ \vdots \\ \nu_v \end{bmatrix} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{processes} \\ \left.\right\} \text{type (here } \textit{read} \text{ or } \textit{write}\text{)} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{input or output value} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{object} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{arbitration relation} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{visibility} \end{array}$$

Encoding histories as words



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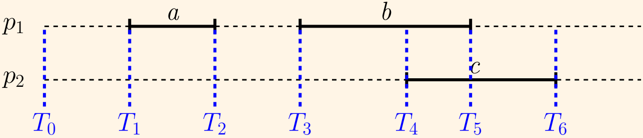

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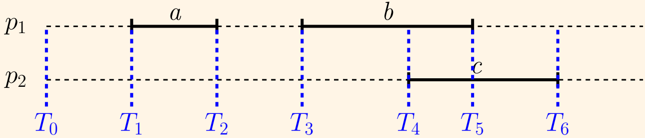
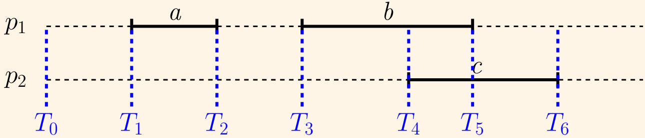


	a	b	c
$proc$	p_1	p_1	p_2
$stime$	T_1	T_3	T_4
$rtime$	T_2	T_5	T_6
$type$	$write$	$read$	$write$
$ival$	14	$\varnothing$	$"fly"$
$oval$	$\varnothing$	$"bee"$	$\varnothing$
obj	y	x	x

Encoding histories as words



	a	b	c
proc	p_1	p_1	p_2
stime	T_1	T_3	T_4
rtime	T_2	T_5	T_6
type	1	0	1
ival	00	$\varnothing$	10
oval	$\varnothing$	01	$\varnothing$
obj	1	0	0

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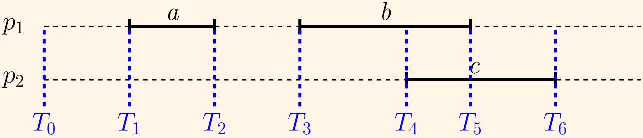

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$$\begin{array}{c} T_0 \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \end{array}$$

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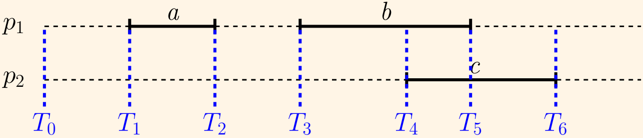

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type	1	0	1
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oval	$\varnothing$	01	$\varnothing$
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$$\begin{array}{cc} T_0 & T_1 \\ \begin{bmatrix}0\\0\\0\\0\\0\\0\\\vdots\end{bmatrix} & \begin{bmatrix}1\\0\\1\\0\\0\\1\\\vdots\end{bmatrix}\end{array}$$

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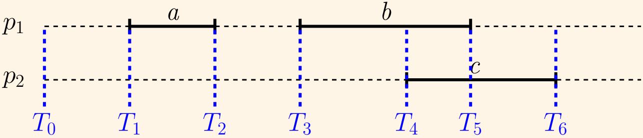

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type	1	0	1
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oval	$\varnothing$	01	$\varnothing$
obj	1	0	0

$$\begin{array}{ccc} T_0 & T_1 & T_2 \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \vdots \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \end{array}$$

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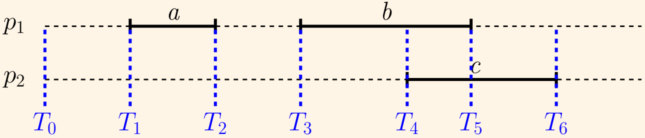

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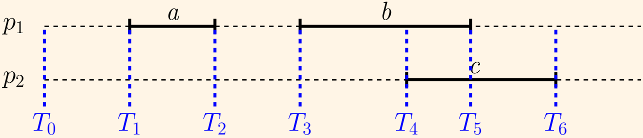

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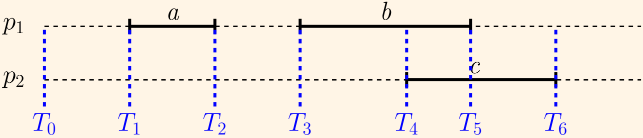

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type	1	0	1
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oval	$\varnothing$	01	$\varnothing$
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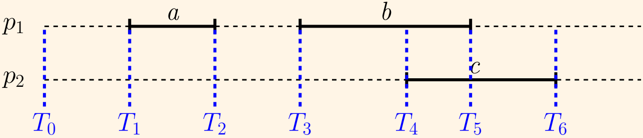
$$
\begin{array}{cccccc}
T_0 & T_1 & T_2 & T_3 & T_4 & T_5 \\
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} &
\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ \vdots \end{bmatrix} &
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} &
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} &
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} &
\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}
\end{array}
$$

Encoding histories as words



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$$\begin{array}{ccccccc} T_0 & T_1 & T_2 & T_3 & T_4 & T_5 & T_6 \\ \begin{bmatrix}0\\0\\0\\0\\0\\0\\\vdots\end{bmatrix} & \begin{bmatrix}1\\0\\1\\0\\0\\1\\\vdots\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\0\\0\\\vdots\end{bmatrix} & \begin{bmatrix}1\\0\\0\\0\\1\\0\\\vdots\end{bmatrix} & \begin{bmatrix}1\\1\\1\\1\\0\\0\\\vdots\end{bmatrix} & \begin{bmatrix}0\\1\\0\\0\\0\\0\\\vdots\end{bmatrix} & \begin{bmatrix}0\\0\\0\\0\\0\\0\\\vdots\end{bmatrix} \end{array}$$

Encoding arbitration

$$
\begin{array}{cc}
 & \begin{array}{ccc} p_1 & \cdots & p_n \end{array} \\
\begin{array}{c} p_1 \\ \vdots \\ p_n \end{array} &
\begin{bmatrix}
0 & \alpha_{12} & \cdots & \alpha_{1n} \\
\neg\alpha_{12} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \alpha_{(n-1)n} \\
\neg\alpha_{1n} & \cdots & \neg\alpha_{(n-1)n} & 0
\end{bmatrix}
\end{array}
$$

A variable α_{ij} is set to 1 if the operation on process i is arbitrated before the operation on process j, and is set to 0 if not

Encoding arbitration

$$
\begin{array}{cc}
 & \begin{array}{ccc} p_1 & \cdots & p_n \end{array} \\
\begin{array}{c} p_1 \\ \vdots \\ p_n \end{array} &
\begin{bmatrix}
0 & \alpha_{12} & \cdots & \alpha_{1n} \\
\neg\alpha_{12} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \alpha_{(n-1)n} \\
\neg\alpha_{1n} & \cdots & \neg\alpha_{(n-1)n} & 0
\end{bmatrix}
\end{array}
\qquad
\begin{bmatrix}
\alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ \vdots \\ \alpha_{23} \\ \alpha_{24} \\ \vdots \\ \alpha_{(n-1)n}
\end{bmatrix}
$$

A variable α_{ij} is set to 1 if the operation on process i is arbitrated before the operation on process j, and is set to 0 if not

Encoding visibility

A way to represent visibility with respect to an operation a, is to encode, for each process p, the most recent operation initiated on p that is visible to a.

$$
\begin{bmatrix} \# \\ \vdots \\ \# \\ p_{1,1} \\ \vdots \\ p_{1,S} \\ \vdots \\ p_{n,1} \\ \vdots \\ p_{n,S} \end{bmatrix}
\begin{array}{l}
\left.\begin{array}{c} \\ \\ \\ \end{array}\right\}\text{previous encoding} \\
\left.\begin{array}{c} \\ \\ \\ \end{array}\right\}\text{most recent operation initiated on } p_1 \text{ that is visible to } a \\
\\
\left.\begin{array}{c} \\ \\ \\ \end{array}\right\}\text{most recent operation initiated on } p_n \text{ that is visible to } a
\end{array}
$$

Compiler

MSO of histories $\rightarrow$ MSO of words of bit vectors

Compiler

MSO of histories → MSO of words of bit vectors

Input:

```
forall X. exists x.  exists y.  x in X and x.type=y.type
```

Output:

```
var2 Process2;
var2 Process1;
var2 TypeRow;
var2 ObjectRow1;
var2 ValueRow1;
pred op(var1 x)= x in Process2 & ex1 y:  y + 1 = x & ~(y in
Process2) | x in Process1 & ex1 y:  y + 1 = x & ~(y in
Process1);
all2 X: ((all1 a:  a in X => op(a)) => ex1 x:  op(x) & ex1
y:  op(y) & x in X & (x in TypeRow <=> y in TypeRow));
```

Conclusion

- MSO satisfiability over finite histories is decidable
- formal and automatic reasoning about consistency models in replicated data systems

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Ongoing work: compiler for translation (in OCaml)

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Future work:

First part: extend to infinite histories (should be easy)

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- ▷ MSO satisfiability over finite histories is decidable
- ▷ formal and automatic reasoning about consistency models in replicated data systems

Ongoing work: compiler for translation (in OCaml)

Future work:

First part: extend to infinite histories (should be easy)

Second part: weaker assumptions (visibility and arbitration)

Thank you